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# Numerical simulation of water entry of twin wedges

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**Abstract**

The hydrodynamic problem of twin wedges entering water vertically at constant speed is analysed based on the velocity potential theory. The gravity effect on the flow is ignored based on the assumption that the ratio of the entry speed to the acceleration due to gravity is much larger than the time scale of interest. The problem is solved using the complex velocity potential together with the boundary element method through three stages. When the body touches water, the similarity solution is obtained for each wedge in isolation. This is used as the initial solution at the second stage for the time stepping technique for each wedge in a stretched system defined through the ratio of the Cartesian system to the distance the wedge travelled into water. When the disturbed zone of each wedge begins to affect the flow generated by the other wedge, the stretched system is abandoned and the original system is used. At the third stage the full interactions between the two wedges are included. Various results are provided for the wave elevation, pressure distribution and force at different deadrise angles. They are compared with those obtained from a single wedge and the interaction effect is investigated.


*Keywords:* Twin wedges; Water entry; Stretches coordinate system

## 1. Introduction

When a body enters water through an initial contact point and subsequently its water plane changes continuously, a unique feature will be that the disturbed fluid domain will increase from zero with the distance the body has travelled in water. The flow at the initial stage is localized in a small area but its structure can be highly complex. In fact, for a wedge with constant speed, the flow is self-similar when the effects due to viscosity and gravity are ignored (Dobrovol'skaya, 1969). This means that the flow pattern at any instant will be the same. Thus at the beginning of the impact, the spatial change of the physical parameters such as pressure and velocity can be extremely rapid in a very small area. Previous numerical work (Zhao and Faltinsen, 1993; Lu et al., 2000) solved this problem by assuming a tiny part of the wedge is already in the water. The solution at the beginning based on this approach is obviously wrong, but it was found that for the constant speed entry of the rigid wedge, the initial error does not affect the solution at a later stage significantly and the solution was found to be in good agreement with the analytical solution. When the wedge is elastic, however, this initial error can be problematic. The error in pressure can lead to the wrong deformation of the wedge. A similar situation is when the wedge enters water through free fall motion (Wu et al., 2004) in which the

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acceleration is unknown. An error in pressure can give the wrong acceleration which gives the wrong velocity. This false information is fed back into the flow through the boundary condition and the final solution could be quite misleading.

To deal with this kind of problem, an ideal numerical approach is to use the stretched coordinate system which is defined through the ratio of the distance in the physical domain to the length travelled by the body into the water (Wu et al., 2004). Although the computational domain will still change due to the deformation of the free surface, the order of its overall size will remain more or less the same together with that of the element, while both of them will vary with the disturbed zone of the free surface in the physical domain. As a result, tiny elements will be used in a very small physical domain at the beginning, and the rapid spatial variation of velocity and pressure can be captured. This technique was used by Wu et al. (2004) and the acceleration of the wedge obtained from the numerical simulation is found to be in good agreement with the experimental data.

Here we consider the water entry problem of twin wedges. One of the obvious applications of this analysis is the slamming loads on a catamaran. The analysis is not just to use a developed code for a single body for a different configuration. In fact, it is necessary to use a three-stage approach here. The first stage is when the body touches water. The flow around each wedge is fully independent. The interaction between the two wedges is wholly negligible and the flow around each wedge can be obtained through the similarity solution. In the second stage, the time stepping method is used and the similarity solution obtained from the first stage is used as the initial solution. The flow around each wedge is, however, still independent and is virtually symmetric about its centreline. In the third stage, the disturbed zones of both wedges begin to meet and their interaction becomes significant. The centreline of each wedge is no longer the symmetry line for the local flow and the two wedges have to be treated together.

Korobkin (1998) considered the water entry of a catamaran section. He included the effect of compressibility of the liquid and the elasticity of the body, but both the free surface and the body surface boundary conditions were linearized and were satisfied approximately. The present analysis is based on the fully nonlinear model and boundary conditions are imposed at their exact locations.

The boundary value problem at each stage is solved using the technique adopted by Wu and Eatock Taylor (1995) and Wu et al. (2004) based on complex velocity potential theory together with the boundary element method. The similarity solution at the first stage is obtained through iteration in a stretched system. Only half of a wedge is considered because the interactions between two wedges are negligible and the flow around each wedge is symmetric. The vertical line below the tip of the wedge is treated as a streamline which is extended to the control boundary far away from the wedge. In this way, the body surface and the control surface far away from the body become a single streamline. At the second stage, the solution is still obtained in the same stretched system through a similar procedure. The difference is that the boundary conditions on the free surface are now satisfied through the time stepping technique as the problem is considered in the time domain. At the third stage, the advantage of the stretched system will disappear, because the distance between the two wedges is time-dependent in this system, while it is in fact constant in the physical domain. Also, when the interactions between the two wedges become important at this stage, the flow around each wedge is no longer symmetric about the centreline of the body. The vertical line below each wedge is no longer a streamline. As a result, the body surface and control surface away from the body will become two separate streamlines, which requires a substantial modification to the complex potential approach used in stages one and two.

Simulations are made for twin wedges with various deadrise angles. Results are provided for pressure, force and wave elevation at different stages.

## 2. Governing equations and numerical procedure

We consider the hydrodynamic problem due to two identical wedges entering the water surface with vertical speed $V$, as shown in Fig. 1. A Cartesian coordinate system $O-xy$ fixed in space is defined in which $y$ points vertically upwards along the centreline of the left wedge and the origin of the system is on the mean free surface. The two wedges form mirror images to each other about $x = L$. The fluid is assumed to be incompressible and inviscid, and the flow is assumed to be irrotational. A velocity potential $\phi$ can then be introduced, which satisfies the Laplace equation

$$\nabla^2\phi = 0 \tag{1}$$

in the fluid domain $R$. On the body surface $S_0$, we have

$$\frac{\partial \phi}{\partial n} = -Vn_y, \tag{2}$$

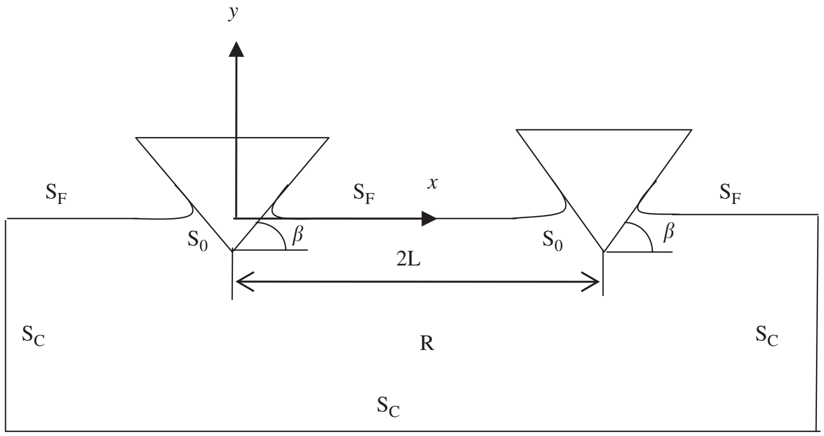



Fig. 1. Sketch of the problem.

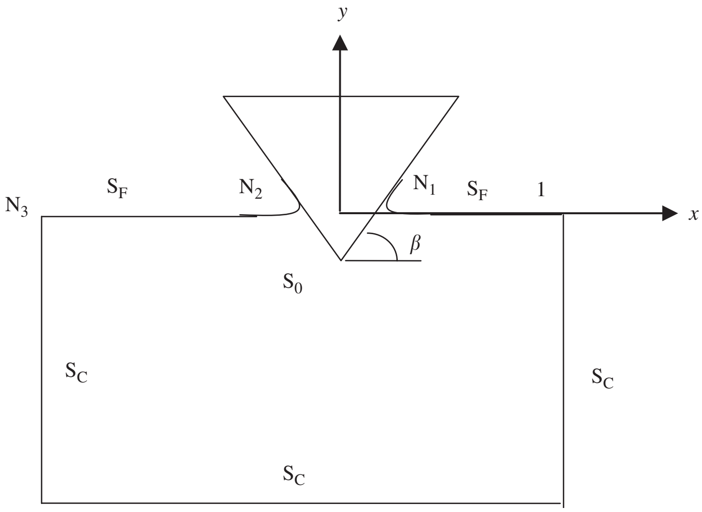



Fig. 2. Computational domain for a single wedge.

where $V$ is positive when the body moves downwards, and $\boldsymbol{n} = (n_x, n_y)$is the normal vector of the body surface pointing out of the fluid domain. The kinematic and dynamic conditions on the free surface $S_F$ or $y = \zeta$ can be written as

$$\frac{\partial \phi}{\partial y} = \frac{\partial \zeta}{\partial t} + \frac{\partial \phi}{\partial x}\frac{\partial \zeta}{\partial x}, \tag{3}$$

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}\nabla\phi\nabla\phi = 0. \tag{4}$$

The effect of acceleration $g$ due to gravity has been ignored in Eq. (4). This is because the main interest in this kind of problem is when the velocity is large and is at the initial period of the impact. In fact, Korobkin and Wu (2000) have discussed that for a floating cylinder moving down suddenly, the effect of $g$ is of order $t^2$ when $t$ is small. In the Lagrangian form, the free surface boundary conditions can be written as

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \phi_t + \nabla\phi\nabla\phi = \frac{1}{2}\nabla\phi\nabla\phi, \tag{5}$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \phi_x, \quad \frac{\mathrm{d}y}{\mathrm{d}t} = \phi_y. \tag{6}$$

The above problem can be solved in general through a time stepping method. At each step, it is convenient to use the complex potential $w = \phi + i\psi$, where $\psi$ is the stream function. The method has been used in a variety of two-dimensional problems (Longuet-Higgins and Cokelet, 1976; Greenhow, 1982; Lin et al., 1985; Wu and Eatock Taylor, 1995; Wu et al., 2004). A brief summary is given here. Cauchy's theorem gives

$$\oint \frac{w}{z - z_0}\,\mathrm{d}z = 0, \tag{7}$$

where $z = x + iy$ and $z_0$ is a point outside of the fluid domain $R$. The integral in Eq. (7) is along the boundary of $R$. Because of symmetry, we only consider half of the domain with $x \leqslant L$. As a result $x = L$ becomes part of $S_C$ and they form a single streamline, as shown in Fig. 2. On the fluid boundary, we can write

$$w = \sum_{j=1}^{n} w_j N_j(z), \tag{8}$$

where $w_j$ is the nodal values of the complex potential and $n$ is the total number of nodes. The interpolation function is chosen as

$$N_j(z) = \begin{cases} (z - z_{j+1})/(z_j - z_{j+1}), & z \in (z_j, z_{j+1}), \\ (z - z_{j-1})/(z_j - z_{j-1}), & z \in (z_{j-1}, z_j), \\ 0, & z \notin (z_{j-1}, z_{j+1}). \end{cases} \tag{9}$$

Substituting Eqs. (8) and (9) into (7), letting $z_0$ approach node $z_k$ and using the boundary conditions, we have

$$\sum_{j=1}^{n} A_{kj}\phi_j|_{j\in S_0+S_C} + i\sum_{j=1}^{n} A_{kj}\psi_j|_{j\in S_F} = -\sum_{j=1}^{n} A_{kj}\phi_j|_{j\in S_F} - i\sum_{j=1}^{n} A_{kj}\psi_j|_{j\in S_0+S_C}, \tag{10}$$

where

$$A_{kj} = \frac{z_k - z_{j-1}}{z_j - z_{j-1}} \ln\frac{z_j - z_k}{z_{j-1} - z_k} + \frac{z_k - z_{j+1}}{z_j - z_{j+1}} \ln\frac{z_{j+1} - z_k}{z_j - z_k}. \tag{11}$$

In Eq. (10), some of the terms have been moved to the right-hand side because $\phi$ on $S_F$ and $\psi$ on $S_0$ and $S_C$ are known at each time step, while those terms on the left are to be found. This procedure is similar to that used in the various previous applications through the complex potential method mentioned above. However, there does exist a major difference here. The control surface $S_C$ is usually treated as a rigid boundary. In the previous applications, it was linked to the body surface to form a single streamline. Here no such link is possible. Thus, if we write $\psi = Vy$ on the body surface, the constant value $C$ of the stream function on $S_C$ cannot be chosen arbitrarily and it should be found from Eq. (10). With the numbering system shown in Fig. 2 we can rewrite Eq. (10) as

$$\sum_{j=N_1+1}^{N_2-1} A_{kj}\phi_j + \sum_{j=N_3+1}^{n} A_{kj}\phi_j + i\sum_{j=2}^{N_1-1} A_{kj}\psi_j + i\sum_{j=N_2+1}^{N_3-1} A_{kj}\psi_j$$
$$+ i\left(A_{k1} + \sum_{n=N_3}^{n} A_{kj}\right)C = -\sum_{j=1}^{N_1} A_{kj}\phi_j - \sum_{j=N_2}^{N_3} A_{kj}\phi_j - i\sum_{j=N_1}^{N_2} A_{kj}\psi_j. \tag{12}$$

Now, when $1 < k < N_1$ or $N_2 < k < N_3$, the real part of this equation will be taken, and when $k = 1$ or $N_3 < k < n$ the imaginary part will be taken. Further, we can choose a node in the latter category and take the real part, which is to obtain an extra equation for $C$. This then leads to the number of equations being equal to the number of unknowns and the problem can be solved.

When the potential has been found, the Bernoulli equation can be used to obtain the pressure

$$p = -\rho\,\phi_t - \tfrac{1}{2}\rho\,\nabla\phi\,\nabla\phi, \tag{13}$$

where $\rho$ is the density of the fluid. As discussed by Wu and Eatock Taylor (2003), $\chi = \phi_t$ is not yet given at each time step; but it can be treated as another harmonic function as it satisfies the Laplace equation. On the free surface, we have

$$\chi = -\tfrac{1}{2}\nabla\phi\,\nabla\phi, \tag{14}$$

because $p = 0$. On the wedge surface, the boundary condition can be written as (Wu, 1998)

$$\frac{\partial\chi}{\partial n} = V\frac{\partial\phi_y}{\partial n}.$$

Once $\chi$ is found, the horizontal and vertical forces $F_x$ and $F_y$ on a single wedge can be found from

$$\begin{pmatrix} F_x \\ F_y \end{pmatrix} = -\rho \int_{S_0} (\phi_t + \tfrac{1}{2}\nabla\phi\nabla\phi) \begin{pmatrix} n_x \\ n_y \end{pmatrix} \mathrm{d}S. \tag{15}$$

As discussed in the Introduction, the above procedure can be problematic in the initial stage when the tips of the twin wedges touch the water. An appropriate approach is to write the above equations in the stretched system. We have

$$\phi(x, y, t) = sV\varphi(\xi, \eta, t), \tag{16}$$

where $\xi = x/s$, $\eta = y/s$ and $s = Vt$. $\varphi(\xi, \eta, t)$ obviously satisfies the Laplace equation in the coordinate system $(\xi, \eta)$. On the free surface, the Lagrangian form of the boundary conditions can be written as

$$\frac{\mathrm{d}(s\xi)}{\mathrm{d}t} = V\varphi_\xi, \quad \frac{\mathrm{d}(s\eta)}{\mathrm{d}t} = V\varphi_\eta, \tag{17}$$

$$\frac{\mathrm{d}(s\varphi)}{\mathrm{d}t} = \frac{V}{2}(\varphi_\xi^2 + \varphi_\eta^2). \tag{18}$$

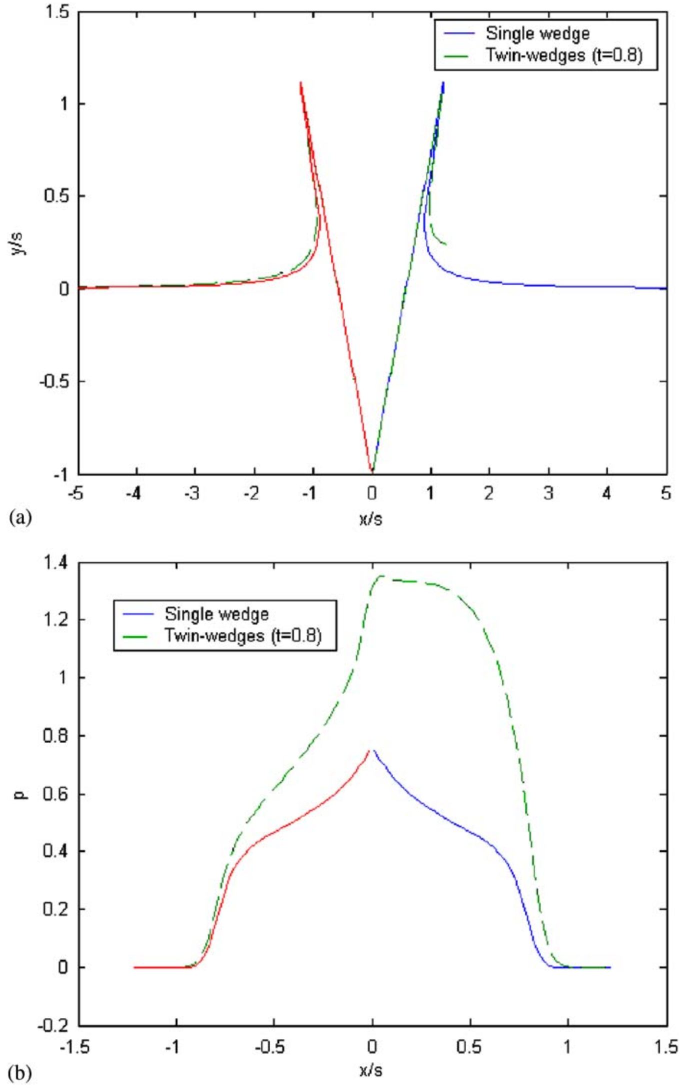



Fig. 3. Results for $\beta = \pi/3$: (a) wave profile, (b) pressure, and (c) force history.

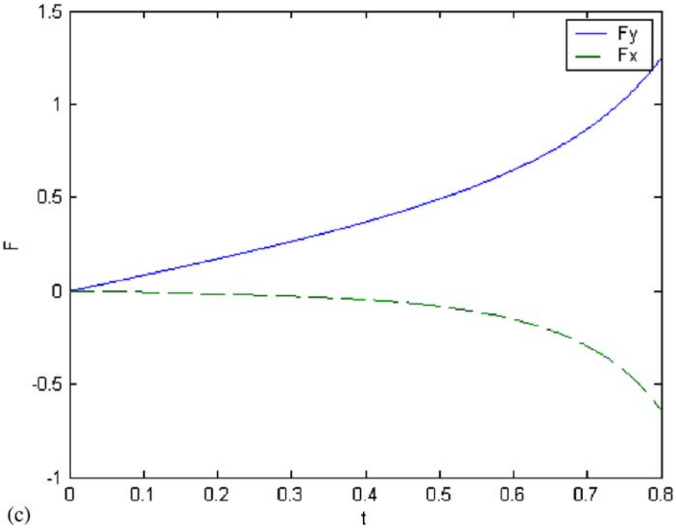
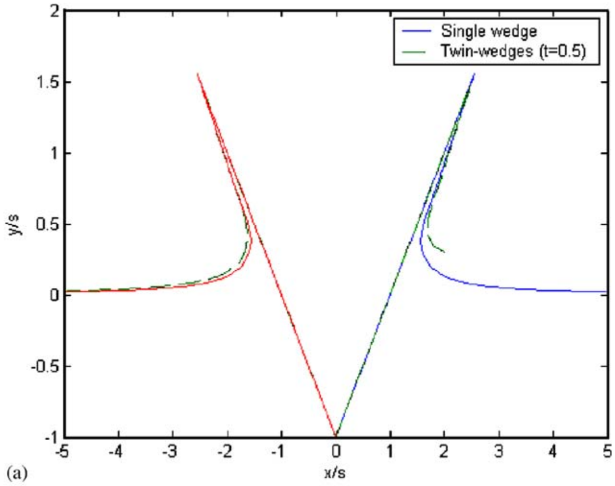



Fig. 3. (*Continued*)



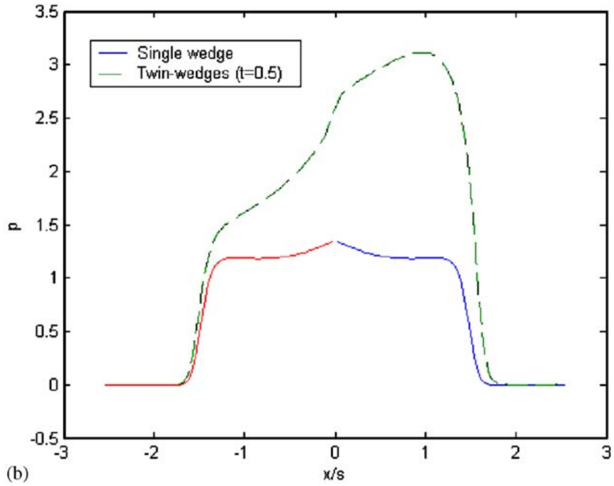
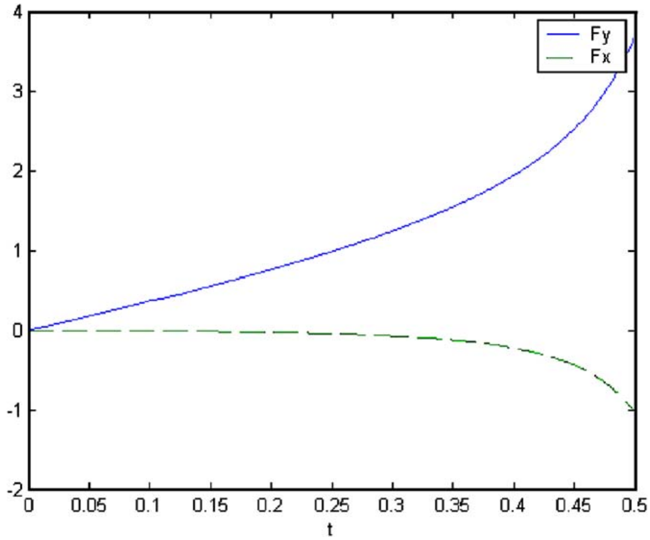
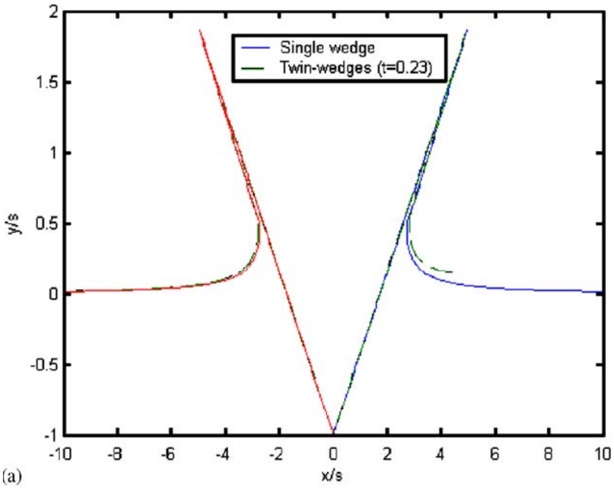




Fig. 4. Results for $\beta = \pi/4$: (a) wave profile, (b) pressure, and (c) force history.

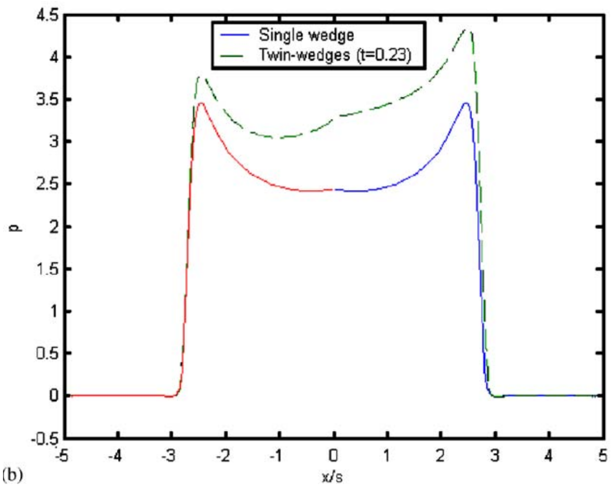



(c)

Fig. 4. (*Continued*)




(a)




(b)

Fig. 5. Results for $\beta = \pi/6$: (a) wave profile, (b) pressure, and (c) force history.

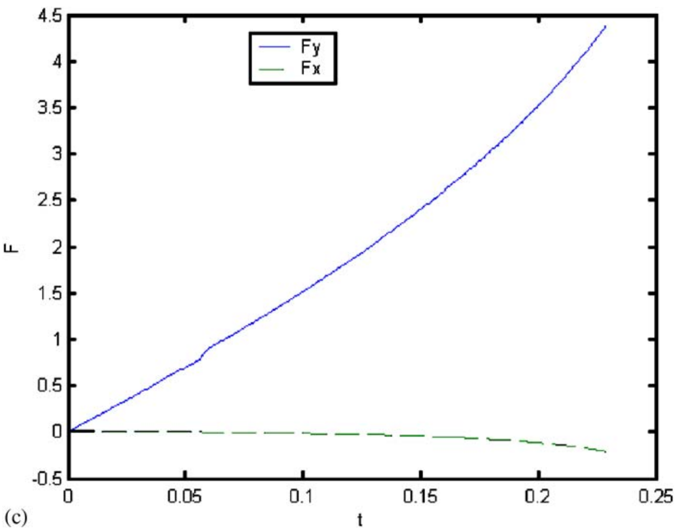



Fig. 5. (*Continued*)

On the body surface, Eq. (2) becomes

$$\frac{\partial \varphi}{\partial n} = -n_\eta. \tag{19}$$

In particular, the two wedges can be treated separately, as the interaction is insignificant. In this case, the vertical line below the tip of the wedge can be treated as the symmetry line for each wedge. It links the body surface and the control surface to form a single streamline. As a result $C$ becomes zero in Eq. (12).

When the time marching technique is used for Eqs. (16)–(19), an initial solution is required. Many analyses used the flat free surface with $\varphi = 0$ as the initial solution. A more rational approach for this problem is to use the similarity solution, which means that Eq. (16) can be written as

$$\phi(x, y, t) = sV\varphi(\xi, \eta). \tag{20}$$

The free surface boundary conditions in Eqs. (3) and (4) can be written as

$$\varphi_\eta - \eta_\xi \varphi_\xi + \xi \eta_\xi - \eta = 0, \tag{21}$$

$$\varphi - \xi \varphi - \eta \varphi_\eta + \tfrac{1}{2}(\varphi_\xi^2 + \varphi_\eta^2) = 0. \tag{22}$$

There is a large number of publications for the similarity problem using various approaches, which include those by Dobrovol'skaya (1969), Howison et al. (1991), Zhao and Faltinsen (1993) and Fraenkel and McLeod (1997). Here the problem is solved by the boundary element method discussed above and the nonlinear boundary conditions on the free surface in Eqs. (21) and (22) are satisfied through an iteration. The details can be found in Wu et al. (2004).

## 3. Results

All the solutions discussed below start from the similarity solution. The time domain method for a single wedge in isolation then takes over. The overall size of the computational domain is more or less fixed in the stretched system $(\xi, \eta)$, but it increases in $(x, y)$ with $s$. When the control surface $S_C$ reaches $x = L$ in the system $(x, y)$, the solution procedure for the twin wedges outlined in Eqs. (1)–(15) will be initiated. The length scale $L$ and the entry speed $V$ are used for nondimensionalization. This means

$$t \to t(L/V), \quad (x, y) \to (x, y)L, \quad p \to p(\rho V^2), \quad (F_x, F_y) \to (F_x, F_y)\rho V^2 L, \quad s \to sL. \tag{23}$$

The elements on the free surface and the body surface used in both stretched system $(\xi, \eta)$ and system $(x, y)$ are uniform. As a Lagrangian method is used to track the free surface, remeshing together with interpolation is applied regularly to avoid over distorted elements. When the jet is developed and it becomes sufficiently thin relative to the element size, it is treated using the shallow water equation in the similarity solution and using Taylor expansion in the time domain solution (Wu et al., 2004). The element size is taken as 0.05 while the time step is taken to ensure the fluid particle will not cross the body surface when the free surface is tracked and its maximum value will not exceed 0.001. Both of these choices have been found to give converged results.

We first consider a case with $\beta = \pi/3$ and results for pressure distribution over the body surface, wave elevation and the force are given in Fig. 3 for the left wedge. The simulation is made until the time when the shortest distance between two bodies at the mean water level is sufficiently small. In fact this can be seen from Fig. 3(a) where the wave elevation for the twin-wedges is truncated at the symmetry line of the two wedges. The figure shows that the wave elevation on the left-hand side of the wedge is not very much affected by the presence of the other wedge. The water level on the right-hand side does rise up compared with the result from the single wedge, but the effect is still nowhere dramatic. The situation with pressure is, however, totally different. Fig. 3(b) shows that the shape of the pressure distribution is completely different when there is another wedge nearby. At $t = 0.8$, the pressure at the tip for the twin-wedges case is almost double that for the single wedge. Fig. 3(c) gives the force history on the wedge. It is evident that the variation is linear initially because the effect by the other wedge is negligible and the solution is similar. When $t$ increases, the variation becomes nonlinear because of interactions between two wedges.

The calculated results for the case with $\beta = \pi/4$ are given in Fig. 4. The overall behaviour of these curves is quite similar to that in Fig. 3. One marked difference is that the point where the highest pressure occurs when $t = 0.5$ has moved towards to right plate of the wedge, which was at the tip in the previous case when $t = 0.5$. Fig. 5 gives the results for the case with $\beta = \pi/6$. Because of the smaller deadrise angle and the longer jet, the simulation stops at $t = 0.23$. The result at this instant shows that the interaction effect on the elevation is even less significant but its effect on the pressure is already quite important.

All these figures have clearly shown the importance of the interaction effect on the pressure. Physically, when the twin wedges enter water, the water between them will be pushed towards the inner surfaces of the wedges. As a result, the pressure will rise together with water level. Fig. 6 gives an evolution of pressure distribution on the wedge with $\beta = \pi/4$. The lowest curve corresponds to $t = 0$, or the moment that the tip of the body touches the water. When the curves rise up, they correspond to $t = 0.1$, 0.2, 0.3, 0.4, 0.5, respectively. The pressure is symmetric initially, but when $t > 0$, asymmetry is developed. It is known that discontinuity of pressure can exist in the inviscid flow (Wu, 2001). It is therefore interesting to see that $p$ seems to remain continuous at $x = 0$, even when the flow is no longer symmetric. This is similar to the case of steady flow over an asymmetric wedge of finite length (Milne-Thomson, 1968). Pressure increases slowly initially in Fig. 6, but when the two wedges are getting very close near the waterline, the pressure goes up very rapidly.

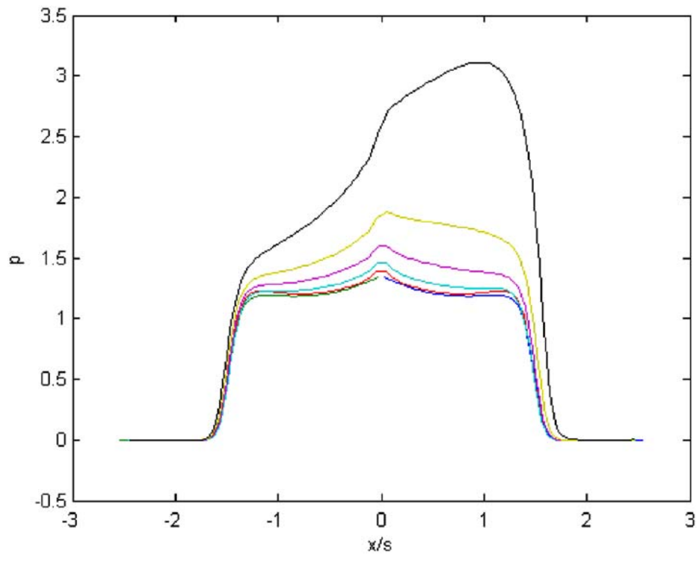



Fig. 6. Evolution of pressure distribution over the wedge with $\beta = \pi/4$.

## 4. Conclusions

The velocity potential problem of twin wedges entering water has been analysed numerically. Different schemes are used at three different stages corresponding to physical features at these stages of the impact. The stability and the accuracy of the results have shown the success of these schemes. The results obtained have shown the interaction effect between two wedges is not highly significant for the wave elevation but it is hugely important for the pressure distribution. The present work can be extended to the case when the body enters through free fall motion (Wu et al., 2004) or when the elasticity of the body needs to be taken into account (Lu et al., 2000).

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